

Non-normalisation de la théorie de Zermelo

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$\neg A$ est défini comme étant $A \rightarrow \perp$; $t \notin s$ signifie $\neg t \in s$; r_a abrège $\{x \in a \mid x \notin x\}$; les coupures sont mises entre []

$$\begin{array}{c}
 \frac{\frac{\frac{r_a \overset{[1]}{\in} r_a \in_E \quad r_a \overset{[1]}{\notin} r_a \rightarrow_E}{\perp} \rightarrow_{I,1}}{[r_a \notin r_a]} \rightarrow_{I,1}}{\perp} \rightarrow_{I,2} \\
 \frac{r_a \overset{[1]}{\in} r_a \in_E \quad r_a \overset{[1]}{\notin} r_a \rightarrow_E}{r_a \in a \quad r_a \notin r_a \in_I} \rightarrow_{I,1} \\
 \frac{r_a \overset{[1]}{\in} r_a \in_E \quad r_a \overset{[1]}{\notin} r_a \rightarrow_E}{r_a \in a \quad r_a \notin r_a \in_I} \rightarrow_{I,1} \\
 \frac{\perp}{r_a \notin a} \rightarrow_{I,2}
 \end{array}
 \iff
 \begin{array}{c}
 \frac{\frac{\frac{r_a \overset{[1]}{\in} r_a \in_E \quad r_a \overset{[1]}{\notin} r_a \rightarrow_E}{\perp} \rightarrow_{I,1}}{r_a \overset{[2]}{\in} a} \in_I}{[r_a \in r_a] \in_E \quad r_a \notin r_a \rightarrow_E} \rightarrow_{I,1} \\
 \frac{\frac{\frac{r_a \overset{[1]}{\in} r_a \in_E \quad r_a \overset{[1]}{\notin} r_a \rightarrow_E}{\perp} \rightarrow_{I,1}}{r_a \overset{[2]}{\in} a} \in_I}{r_a \in a \quad r_a \notin r_a \in_I} \rightarrow_{I,1} \\
 \frac{\perp}{r_a \notin a} \rightarrow_{I,2}
 \end{array}$$

Règles de déduction naturelle utilisées

$$\begin{array}{c} \vdots \\ \vdots \\ t \in s \quad A[x := t] \\ \hline t \in \{x \in s \mid A\} \end{array} \in_I \qquad
 \begin{array}{c} \vdots \\ t \in \{x \in s \mid A\} \\ \hline A[x := t] \end{array} \in_E \qquad
 \begin{array}{c} [1] \\ A \\ \vdots \\ B \\ \hline A \rightarrow B \end{array} \rightarrow_{I,1} \qquad
 \begin{array}{c} \vdots \\ A \rightarrow B \\ \hline B \end{array} \rightarrow_E \qquad
 \begin{array}{c} \vdots \\ A \\ \hline B \end{array} \rightarrow_E$$

Règles d'élimination des coupures utilisées

$$\begin{array}{c} \vdots \\ \vdots \\ t \in s \quad A[x := t] \\ \hline t \in \{x \in s \mid A\} \\ \hline A[x := t] \end{array} \in_I \in_E \quad \Rightarrow \quad \begin{array}{c} \vdots \\ A[x := t] \end{array} \qquad
 \begin{array}{c} [1] \\ A \\ \vdots \\ B \\ \hline A \rightarrow B \end{array} \rightarrow_{I,1} \qquad
 \begin{array}{c} \vdots \\ A \\ \hline B \end{array} \rightarrow_E \quad \Rightarrow \quad \begin{array}{c} \vdots \\ A \\ \hline B \end{array}$$